Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

order of $s_1 s_2$ cannot be less than p^{α_1} nor greater than p^{α}. If we assign to k the following values:

$$p^{\alpha-\alpha_1}-1,\ p^{\alpha-\alpha_1-1}-1,\ p^{\alpha-\alpha_1-2}-1,\ \ldots . p-1,\ 1\ (\alpha_1 \text{ does not equal } \alpha),$$

we observe that $s_1 s_2$ may have for its order any power of p from p^{α_1} to p^{α} when p is odd (or when $p=2$ and $\alpha=\alpha_1$), and from p^{α_1} to $p^{\alpha-1}$ when $p=2$ and $\alpha>\alpha_1$.

These results are expressed by the following

THEOREM. *It is possible to find two commutative operators* (s_1, s_2) *of the same order* (p^{α}) *such that* $s_1 p^{\alpha_1}=s_2 k p^{\alpha_1}$ ($\alpha\overset{=}{<}\alpha$) *and* $s_1 s_2$ *is of order* p^{δ}, *where* δ *can have any value from* α_1 *to* α *when* $p>2$ (*or when* $p=2$ *and* $\alpha=\alpha_1$), *and from* α_1 *to* $\alpha-1$ *when* $p=2$ and $\alpha>\alpha_1$.

When the orders of s_1 and s_2 are not powers of the same prime they may be represented by $2^{\alpha_0}p_1{}^{\alpha_1}p_2{}^{\alpha_2}\ldots .$ and $2^{\beta_0}p_1{}^{\beta_1}p_2{}^{\beta_2}\ldots .$ respectively; p_1, p_2, ... being odd prime numbers, and the exponents being positive integers including 0. We may think of s_1 and s_2 as the products of operators of orders 2^{α_0}, $p_1{}^{\alpha_1}$, $p_2{}^{\alpha_2}$, ... and of orders 2^{β_0}, $p_1{}^{\beta_1}$, $p_2{}^{\beta_2}$, ... respectively. $s_1 s_2$ is then the product of all of these operators. Combining the pairs which are powers of the same prime, $s_1 s_2$ may be represented as the product of operators of orders 2^{γ_0}, $p_1{}^{\gamma_1}$, $p_2{}^{\gamma_2}$, ... ($p_x{}^{\gamma_x}$ being the product of the given operators of orders $p_x{}^{\alpha_x}$ and $p_x{}^{\beta_x}$). Since the groups generated by these operators have only identity in common $s_1 s_2$ is of order $2^{\gamma_0}p_1{}^{\gamma_1}p_2{}^{\gamma_2}\ldots .$ We have now reduced this case to one of the earlier ones.

It was observed in the second paragraph that γ_x is equal to the larger of the two numbers α_x and β_x whenever these are different, and that γ_x may be 0 whenever $\alpha_x=\beta_x$. *Hence the minimum order* (m) *of* $s_1 s_2$ *is the product of the highest of the primes which divide one and only one of their orders.** The maximum order (M) of $s_1 s_2$ is clearly the lowest common multiple of their orders. From the given theorem it follows that s_1 and s_2 can be so selected that the order of $s_1 s_2$ is any factor of M which is not less than m.

◆

HYDRAULIC SOLUTION OF AN ALGEBRAIC EQUATION OF THE nth DEGREE.

By DR. ARNOLD EMCH, University of Colorado.

In the January (1901) number of this MONTHLY I have established two methods of extracting the nth root of any positive real number. In conclusion, I proposed to apply the first (hydrostatic) method to the solution of an equation of the form

$$a_0+a_1x+a_2x^2+\ldots .+a_nx^n=0\ldots .(1).$$

---

*Mr. Fite first called my attention to this minimum value.

It is as follows: Take n solids of revolution corresponding to the volumes x, x^2, $x^3 \ldots . x^n$ and attach them to a lever which may turn about a fixed edge H. The axes of these solids must be kept in a vertical position. Let a_1, a_2, $a_3 \ldots . a_n$ be the distances from H of the points where these axes intersect the lever. Attach also a weight 1 to the lever at a distance $-a_0$ from H. According as a_k is positive or negative attach the corresponding solid to the right or left of H. The lowest points of the solids are all in the same straight line. The whole apparatus is now put in a vessel containing water and the level of water is raised until the lever and with it the lowest points of the solids are all in a horizontal position. In this position the depths of immersion of the solids are all equal to a certain length y and the quantities of water displaced are y, y^2, $y^3 \ldots . y^n$. These act upon the lever as vertical forces in an upward direction.

The moment of all these forces is therefore

$$a_0+a_1y+a_2y^2+\ldots . +a_ny^n \ldots . (2).$$

The other forces acting upon the lever are the weights of the solids themselves. If these are all equal to w, their moment is

$$w(a_1+a_2+a_3+\ldots . +a_n), \text{ hence}$$

$$a_0+a_1y+a_2y^2+\ldots . +a_ny^n+w(a_1+a_2+a_3+ \ldots +a_n)=0 \ldots \ (3).$$

Adding now a weight w at a distance $-(a_1+a_2+a_3+\ldots . +a_n)$ from H and adjusting the water-level in such a manner that the lever assumes again a horizontal position, then the depth of immersion will be x, since in this case

$$a_0+a_1x+a_2x^2+\ldots . +a_nx^n=0 \ldots . (4).$$

Hence, the depth of immersion of the solids is a positive real root of (1).

After the first root has been found, the water-level in the vessel may be raised still higher. Equilibrium will first be destroyed, but at a certain point of the level, equilibrium is again restored and the corresponding depth of immersion will give another positive real root of (1). By this process all positive real roots of the proposed equation may be obtained.

I shall not enter into further details of this interesting problem. Since the publication of my first article on this subject I have found that M. Georges Meslin has published a similar method in the *Journal de Physique* for June, 1900, Vol. IX, III series, page 339. The reader is referred to Professor Meslin's valuable article, where a description of an apparatus for the solution of equations (1) may be found.

Boulder, Col., February 9, 1901.